

# Five-body cluster structure of double- $\Lambda$ hypernucleus ${}^{11}_{\Lambda\Lambda}\text{Be}$

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Energy levels of the double  $\Lambda$  hypernucleus,  ${}^{11}_{\Lambda\Lambda}\text{Be}$  are calculated within the framework of an  $\alpha\alpha n\Lambda\Lambda$  five-body model. Interactions between constituent particles are determined so as to reproduce reasonably the observed low-energy properties of the  $\alpha\alpha$ ,  $\alpha\alpha n$  nuclei and the existing data for  $\Lambda$ -binding energies of the  $\alpha\Lambda$ ,  $\alpha\alpha\Lambda$ ,  $\alpha n\Lambda$  and  $\alpha\alpha n\Lambda$  systems. An effective  $\Lambda\Lambda$  interaction is constructed so as to reproduce, within the  $\alpha\Lambda\Lambda$  three-body model, the  $B_{\Lambda\Lambda}$  of  ${}^6_{\Lambda\Lambda}\text{He}$ , which was extracted from the emulsion experiment, the NAGARA event. With no adjustable parameters for the  $\alpha\alpha n\Lambda\Lambda$  system,  $B_{\Lambda\Lambda}$  of the ground and bound excited states of  ${}^{11}_{\Lambda\Lambda}\text{Be}$  are calculated with the Gaussian Expansion Method. The Hida event, recently observed at KEK-E373 experiment, is interpreted as an observation of the ground state of the  ${}^{11}_{\Lambda\Lambda}\text{Be}$ .

In nuclear physics involving strangeness, the fundamental problem is to describe the different facets of the interactions among the baryon octet in a unified way. Our aim is to reveal various features of hyperon( $Y$ )-nucleon( $N$ ) and  $YY$  interactions through combined analyses for two- and many-body hyperonic systems. For instance,  $\Lambda N$  interaction models have been constructed so far by utilizing various  $\Lambda$  hypernuclear data to complement the limited  $\Lambda N$  scattering data. The  $\Lambda\Lambda$  interaction is an important entry into the  $S = -2$  baryon-baryon interactions, decisive information about which is obtained from observations of double- $\Lambda$  hypernuclei and their separation energies for two  $\Lambda$ 's separated from a double- $\Lambda$  hypernucleus, denoted as  $B_{\Lambda\Lambda}$ .

In the KEK-E176/E373 hybrid emulsion experiments, there were observed several events corresponding to double- $\Lambda$  hypernuclei. Among them was the epoch-making observation of the NAGARA event, which was identified uniquely as  ${}^6_{\Lambda\Lambda}\text{He}$  in the ground state with a precise value of  $B_{\Lambda\Lambda} = 6.91 \pm 0.16$  MeV [1, 2]. A second important observation was the Demachi-Yanagi event [2, 3] identified as  ${}^{10}_{\Lambda\Lambda}\text{Be}$  with  $B_{\Lambda\Lambda} = 11.90 \pm 0.13$  MeV [13] though it was uncertain whether this event was interpreted to be the ground state or an excited state.

A newly observed double- $\Lambda$  event has been recently reported, called the Hida event [2]. This event has two possible interpretations: One is  ${}^{11}_{\Lambda\Lambda}\text{Be}$  with  $B_{\Lambda\Lambda} = 20.83 \pm 1.27$  MeV, and the other is  ${}^{12}_{\Lambda\Lambda}\text{Be}$  with  $B_{\Lambda\Lambda} = 22.48 \pm 1.21$  MeV. It is uncertain whether this is an observation of a ground state or an excited state.

In the planned experiments at J-PARC, dozens of emulsion events for double- $\Lambda$  hypernuclei will be produced. In emulsion experiments, however, it is difficult to determine the spin-parity or even to know whether an observed event corresponds to a ground or excited state. Therefore, it is vitally important to compare the emulsion data with theoretical analyses to obtain a proper interpretation.

In order to interpret the 'Demachi-Yanagi' event, we studied in Ref.[3] the  ${}^{10}_{\Lambda\Lambda}\text{Be}$  hypernucleus within the framework of the  $\alpha\alpha\Lambda\Lambda$  four-body cluster model, where

the  $\Lambda\Lambda$  interaction was taken consistently with the NAGARA event. Our calculated value for the  $2^+$  state was in good agreement with the observed data. Thus, the Demachi-Yanagi event is interpreted as an observation of the  $2^+$  excited state of  ${}^{10}_{\Lambda\Lambda}\text{Be}$ .

The aim of this paper is to interpret the new Hida event on the basis of our theoretical study, adapting the method used for interpreting the Demachi-Yanagi event. At present, the Hida event has two possible interpretations:  ${}^{11}_{\Lambda\Lambda}\text{Be}$  and  ${}^{12}_{\Lambda\Lambda}\text{Be}$ . In this paper, we assume this event is a  ${}^{11}_{\Lambda\Lambda}\text{Be}$  hypernucleus. It is reasonable to employ an  $\alpha\alpha n\Lambda\Lambda$  five-body model for the study of  ${}^{11}_{\Lambda\Lambda}\text{Be}$ , because, as mentioned above, the interpretation of the Demachi-Yanagi event for  ${}^{10}_{\Lambda\Lambda}\text{Be}$  was possible on the basis of an  $\alpha\alpha\Lambda\Lambda$  four-body cluster model, and  ${}^{11}_{\Lambda\Lambda}\text{Be}$  is composed of  ${}^{10}_{\Lambda\Lambda}\text{Be}$  plus one additional neutron. We further note that the core nucleus  ${}^9\text{Be}$  is well described by using an  $\alpha\alpha n$  three-cluster model [4], and, therefore, it should be possible to model the structure change of  ${}^9\text{Be}$  due to the addition of the two  $\Lambda$  particles as a five-body problem.

The  $\alpha\alpha n\Lambda\Lambda$  five-body cluster model employed in this paper is quite challenging as a numerical computation, because of the following conditions: (1) there exist three species of particles ( $\alpha$ ,  $\Lambda$ , and neutron), (2) five different kinds of interactions ( $\Lambda$ - $\Lambda$ ,  $\Lambda$ -neutron,  $\Lambda$ - $\alpha$ , neutron- $\alpha$ , and  $\alpha$ - $\alpha$ ) are involved, (3) one must take into account the Pauli principle between the two  $\alpha$  particles and between the  $\alpha$  and neutron. We have succeeded in developing our Gaussian Expansion Method [5], used in the above mentioned study of  ${}^{10}_{\Lambda\Lambda}\text{Be}$ , in order to perform this five-body cluster-model calculation.

It should be emphasized, before going to the five-body calculation, that all the interactions are determined so as to reproduce the observed binding energies of the two- and three-body subsystems ( $\alpha\alpha$ ,  $\alpha n$ ,  $\alpha\alpha n$ ,  $\alpha\Lambda$ ,  $\alpha\alpha\Lambda$ ,  $\alpha\Lambda\Lambda$ , and  $\alpha n\Lambda$ ). We then calculated the energies of the  $A = 10$  four-body subsystems,  ${}^{10}_{\Lambda}\text{Be}$  ( $= \alpha\alpha n\Lambda$ ) and  ${}^{10}_{\Lambda\Lambda}\text{Be}$  ( $= \alpha\alpha\Lambda\Lambda$ ), and found that they are simultaneously reproduced with no additional adjustable parameters needed. We are then able to calculate both the

ground and excited states of  $^{11}_{\Lambda\Lambda}\text{Be}$  with no free parameter. On the basis of careful calculations, we shall show that the recently reported Hida event can be considered to be an observation of the ground state of  $^{11}_{\Lambda\Lambda}\text{Be}$ .

In order to take into account the full five-body degrees of freedom of the  $\alpha\alpha n\Lambda\Lambda$  system and the full correlations among all the constituent five particles, we describe the total wave function,  $\Psi_{JM}(^{11}_{\Lambda\Lambda}\text{Be})$ , as a function of the entire 35 sets of Jacobi coordinates  $\{\mathbf{r}_c, \mathbf{R}_c, \boldsymbol{\rho}_c, \mathbf{S}_c; c = 1 - 35\}$  in which the two  $\Lambda$ 's (two  $\alpha$ 's) have been antisymmetrized (symmetrized). Some of the important coordinate sets ( $c = 1 - 6$ ) are shown in Fig. 1.

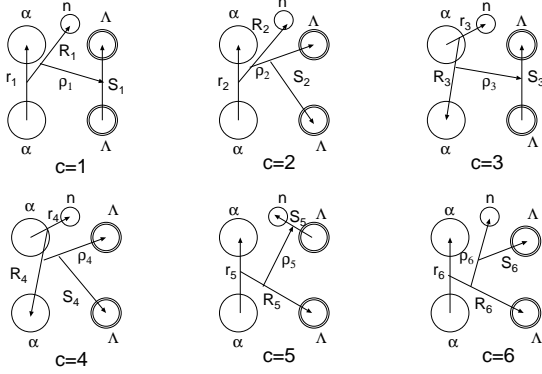


FIG. 1: Example sets ( $c = 1 - 6$ ) of Jacobi coordinates of the  $\alpha\alpha n\Lambda\Lambda$  five-body system. Antisymmetrization (symmetrization) of two  $\Lambda$ 's (two  $\alpha$ 's) is to be made.

The total wave function is described as a sum of five-body basis functions  $\Phi_{JM,\beta}^{(c)}$ , which is an extension of our previous work using four-body basis functions [6]:

$$\Psi_{JM}(^{11}_{\Lambda\Lambda}\text{Be}) = \sum_{c=1}^{35} \sum_{\beta} C_{\beta}^{(c)} \mathcal{A}_{\Lambda\Lambda} \mathcal{S}_{\alpha\alpha} \Phi_{JM,\beta}^{(c)}, \quad (1)$$

where  $\mathcal{A}_{\Lambda\Lambda}$  ( $\mathcal{S}_{\alpha\alpha}$ ) is the antisymmetrizer (symmetrizer) for two  $\Lambda$ 's (two  $\alpha$ 's), and

$$\begin{aligned} \Phi_{JM,\beta}^{(c)} &= \xi(\alpha_1)\xi(\alpha_2) \\ &\times \left[ \left[ \left[ \left[ \chi_{nl}^{(c)}(\mathbf{r}_c) \psi_{NL}^{(c)}(\mathbf{R}_c) \right]_I \varphi_{n'l'}^{(c)}(\boldsymbol{\rho}_c) \right]_K \Phi_{N'L'}^{(c)}(\mathbf{S}_c) \right]_L \right. \\ &\times \left. \left[ \left[ \chi_{\frac{1}{2}}(\Lambda_1) \chi_{\frac{1}{2}}(\Lambda_2) \right]_{\Sigma} \chi_{\frac{1}{2}}(n) \right]_S \right]_{JM}, \quad (2) \end{aligned}$$

with  $\beta \equiv \{nl, NL, n'l', N'L', IKL, \Sigma S\}$  denoting a set of the quantum numbers. In Eq. (2),  $\xi(\alpha)$  is the internal wave function of an  $\alpha$ -cluster having  $(0s)^4$  configuration and is used in the folding procedures for the  $\alpha n$ ,  $\alpha\Lambda$ , and  $\alpha\alpha$  interactions. The  $\chi_{\frac{1}{2}}(\Lambda)$  and  $\chi_{\frac{1}{2}}(n)$  are the spin functions of the  $\Lambda$  and  $n$ , respectively. Following Refs. [5, 6], the radial shapes of the basis function  $\phi_{nlm}(\mathbf{r}) (= r^l e^{-(r/r_n)^2} Y_{lm}(\hat{\mathbf{r}}))$  are taken to be Gaussians

with ranges postulated to lie in a geometric progression and similarly for  $\psi_{NLM}(\mathbf{R})$ ,  $\varphi_{n'l'm'}(\boldsymbol{\rho})$  and  $\Phi_{N'L'M'}(\mathbf{S})$ . The expansion coefficients  $C_{\beta}^{(c)}$  and the eigenenergy  $E$  of the total wave function  $\Psi_{JM}(^{11}_{\Lambda\Lambda}\text{Be})$  are determined by solving the five-body Schrödinger equation using the Rayleigh-Ritz variational method.

In the present  $\alpha\alpha n\Lambda\Lambda$  five-body model for  $^{11}_{\Lambda\Lambda}\text{Be}$ , it is absolutely necessary that any subcluster systems composed of the two, three, or four constituent particles are reasonably described by taking the interactions among these systems. In our previous work on double  $\Lambda$  hypernuclei with  $A = 7 - 10$  within the framework of the  $\alpha x\Lambda\Lambda$  four-body cluster model ( $x = n, p, d, t, {}^3\text{He}$ , and  $\alpha$ ) [6], the  $\alpha\alpha$ ,  $\alpha n$ ,  $\alpha\Lambda$ ,  $\Lambda n$ , and  $\Lambda\Lambda$  interactions were determined so as to reproduce well the following observed quantities: (i) Energies of the low-lying states and scattering phase shifts in the  $\alpha n$  and  $\alpha\alpha$  systems, (ii)  $\Lambda$ -binding energies  $B_{\Lambda}$  in  ${}^5_{\Lambda}\text{He}$  ( $= \alpha\Lambda$ ),  ${}^6_{\Lambda}\text{He}$  ( $= \alpha n\Lambda$ ) and  ${}^9_{\Lambda}\text{Be}$  ( $= \alpha\alpha\Lambda$ ), (iii) double- $\Lambda$  binding energies  $B_{\Lambda\Lambda}$  in  ${}^6_{\Lambda\Lambda}\text{He}$  ( $= \alpha\Lambda\Lambda$ ), the NAGARA event. We then predicted, with no more adjustable parameters, the energy level of  $^{10}_{\Lambda\Lambda}\text{Be}$  ( $= \alpha\alpha\Lambda\Lambda$ ), and found that, as mentioned before, the Demachi-Yanagi event was an observation of the  $2^+$  excited state of  $^{10}_{\Lambda\Lambda}\text{Be}$ .

In the present five-body calculation, we employ the interactions of Ref. [6] so that those severe constraints are also successfully met in our two-, three-, and four-body subsystems. But, the present core nucleus  ${}^9\text{Be}$  ( $= \alpha\alpha n$ ) does not belong to the subsystems studied previously. Since use of the same  $\alpha\alpha$  and  $\alpha n$  interactions does not precisely reproduce the energies of the low-lying states of  ${}^9\text{Be}$  measured from the  $\alpha\alpha n$  threshold (the same difficulty is seen in another microscopic  $\alpha\alpha n$  cluster-model study [4]), we introduce an additional phenomenological  $\alpha\alpha n$  three-body force with a Gaussian shape,  $v_0 e^{-(r_{\alpha-\alpha}/r_0)^2 - (R_{\alpha\alpha-n}/R_0)^2}$ , having  $r_0 = 3.6$  fm,  $R_0 = 2.0$  fm and  $v_0 = -9.7$  MeV (+13.0 MeV) for the negative-parity (positive-parity) state so as to fit the observed energies of the  $3/2_1^-$  ground state and the  $5/2_1^-, 1/2_1^-$  and  $1/2_1^+$  excited states in  ${}^9\text{Be}$ . For the latter three resonance states, the same bound-state approximation (namely, diagonalization of the Hamiltonian with the  $L^2$ -integrable basis functions) was applied. Simultaneously, we found that the calculated  $B_{\Lambda}$  of the ground state ( $1^-$ ) of  $^{10}_{\Lambda}\text{Be}$  reproduced well the observed value.

In order to reproduce the observed  $B_{\Lambda\Lambda} = 6.91$  MeV for  ${}^6_{\Lambda\Lambda}\text{He}$ , we tuned the  $\Lambda\Lambda$  interaction {Eq.(3.6) in Ref. [6]} by multiplying the strength of the  $i = 3$  part by a factor 1.244. As for  $^{10}_{\Lambda\Lambda}\text{Be}$ , we obtained  $B_{\Lambda\Lambda}^{\text{cal}}(2_1^+) = 11.88$  MeV and  $B_{\Lambda\Lambda}^{\text{cal}}(0_1^+) = 14.74$  MeV, which explain, respectively, the Demachi-Yanagi event for the  $2_1^+$  state ( $11.90 \pm 0.13$  MeV) and the data of Ref. [8] for the ground state ( $14.6 \pm 0.4$  MeV; see Table 5 of Ref.[8]). The above successful check of the energies of the subsystems encourages us to perform the five-body calculation of  $^{11}_{\Lambda\Lambda}\text{Be}$  with no adjustable parameter, expecting high reliability for the result.

In Fig. 2, convergence of the calculated energy of

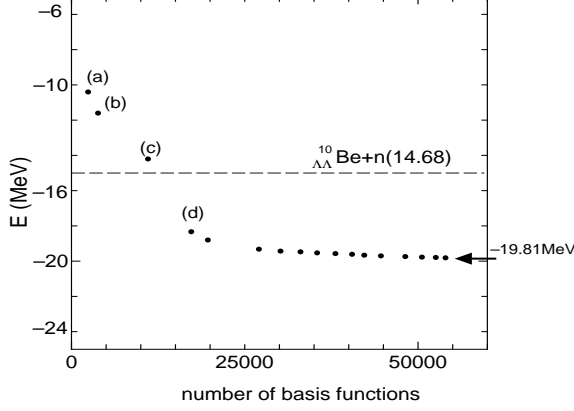


FIG. 2: Convergence of the energy  $E$  of the  $3/2^-$  ground state in  $^{11}_{\Lambda\Lambda}\text{Be}$  with respect to increasing number of the  $\alpha\alpha n\Lambda\Lambda$  five-body basis functions. The energy is measured from the five-body breakup threshold. The dashed line shows the lowest threshold,  $^{10}_{\Lambda\Lambda}\text{Be} + n$  threshold. See text for points (a)-(d).

the  $3/2^-$  ground state of  $^{11}_{\Lambda\Lambda}\text{Be}$  is illustrated with increasing number of five-body basis functions  $\Phi_{JM,\beta}^{(c)}$  in Eq.(1). The energy is measured from the five-body breakup threshold. The lowest threshold is  $^{10}_{\Lambda\Lambda}\text{Be} + n$ , located at  $-14.68$  MeV. The most important configuration in the ground-state wave function is found to be of the  $^9\text{Be}^* + \Lambda + \Lambda$  type, namely the configuration described by using the Jacobi-coordinate sets  $c = 1 - 4$  in Fig. 1; here,  $^9\text{Be}^*$  denotes the  $\alpha\alpha n$  three-body degrees of freedom. The energy point (a) is obtained by taking  $c = 1$  configuration in Fig.1, the point (b) is obtained by  $c = 1$  and 2 and the point (c) is obtained by  $c = 1$  to 4. The additional 4-MeV energy gain from point (c) to (d) is obtained by including the configurations of the  $^9_{\Lambda}\text{Be}^* + n + \Lambda$  type described with the Jacobi coordinates such as  $c = 5$  and 6, in which  $^9_{\Lambda}\text{Be}^*$  stands for the  $\alpha\alpha\Lambda$  degrees of freedom. Another 1.5-MeV gain from point (d) down to the converged value ( $-19.81$  MeV with the accuracy of 10 keV totally with  $\sim 50,000$  basis functions) is achieved by including all the other types of configurations such as  $^{10}_{\Lambda\Lambda}\text{Be}^* + n$ ,  $^{10}_{\Lambda}\text{Be}^* + \Lambda$ ,  $^{7}_{\Lambda\Lambda}\text{He}^* + \alpha$ ,  $^6_{\Lambda}\text{He}^* + ^5_{\Lambda}\text{He}^*$ . Thus, the ground state is found to be bound by 5.1 MeV below the lowest threshold. The angular momentum space of  $l, l', L, L', \lambda \leq 2$  was found to be sufficient to obtain convergence for the energy.

TABLE I: Calculated r.m.s. distances  $\bar{r}_{\alpha-\alpha}$ ,  $\bar{r}_{\alpha-n}$  and  $\bar{r}_{(\alpha\alpha)-n}$  in  $^9\text{Be}$ ,  $^{10}_{\Lambda}\text{Be}$  and  $^{11}_{\Lambda\Lambda}\text{Be}$ .

(fm)	$\bar{r}_{\alpha-\alpha}$	$\bar{r}_{\alpha-n}$	$\bar{r}_{(\alpha\alpha)-n}$
$^9\text{Be}$	3.68	3.98	3.54
$^{10}_{\Lambda}\text{Be}$	3.28	3.53	3.14
$^{11}_{\Lambda\Lambda}\text{Be}$	3.10	3.33	2.94

It is interesting to look at the dynamical change of the nuclear core,  $^9\text{Be}$ , which occurs due to the addition of two  $\Lambda$  particles. The possibility of nuclear-core shrinkage due to a  $\Lambda$ -particle addition was originally pointed out in Ref. [9] by using the  $\alpha x\Lambda$  three-cluster model ( $x = n, p, d, t, ^3\text{He}$ , and  $\alpha$ ) for  $p$ -shell  $\Lambda$  hypernuclei. As for the hypernucleus  $^7_{\Lambda}\text{Li}$ , the prediction of some 20%-shrinkage, in Ref. [9] and in an updated calculation [10], was actually confirmed by experiment [11]. As far as  $p$ -shell double- $\Lambda$  hypernuclei are concerned, the present authors found [6], using an  $\alpha x\Lambda\Lambda$  model, that participation of the second  $\Lambda$  particle can induce a further  $\sim 8\%$  reduction in the distance between the  $\alpha$  and  $x$  in the nuclear core. For the present five-body  $\alpha\alpha n\Lambda\Lambda$  system,  $^{11}_{\Lambda\Lambda}\text{Be}$ , Table I shows the r.m.s. distances  $\bar{r}_{\alpha-\alpha}$  between two  $\alpha$  particles,  $\bar{r}_{\alpha-n}$  between  $\alpha$  and  $n$  and  $\bar{r}_{(\alpha\alpha)-n}$  between  $(\alpha\alpha)$  and  $n$  in  $^9\text{Be}$ ,  $^{10}_{\Lambda}\text{Be}$  and  $^{11}_{\Lambda\Lambda}\text{Be}$ . When the first  $\Lambda$  particle is added to  $^9\text{Be}$ , a reduction of those distances is about 20%, whereas the adding a second  $\Lambda$  reduces the distances by about 6%; this is similar to the case of double  $\Lambda$  hypernuclei studied with the  $\alpha x\Lambda\Lambda$  four-body model [6].

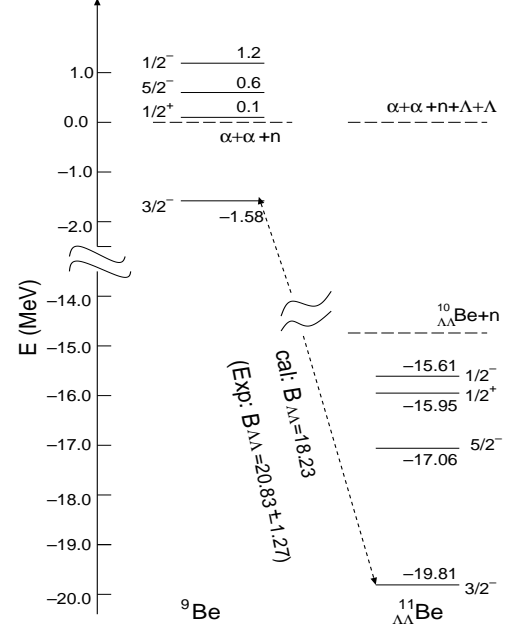


FIG. 3: Calculated energy spectra of the low-lying states of  $^{11}_{\Lambda\Lambda}\text{Be}$  together with those of the core nucleus  $^9\text{Be}$ .

Finally, let us discuss the energy spectra of  $^{11}_{\Lambda\Lambda}\text{Be}$  and its relation to the Hida event. Using the same framework and interactions as those in the  $3/2^-$  ground state, we calculated energies and wave functions of the  $5/2^-$ ,  $1/2^+$  and  $1/2^-$  states of  $^{11}_{\Lambda\Lambda}\text{Be}$ ; there is no other bound state below the lowest  $^{10}_{\Lambda\Lambda}\text{Be} + n$  threshold. The energy level is illustrated in Fig. 3 together with that of  $^9\text{Be}$ . Interestingly enough, the order of the  $1/2^+$  and  $5/2^-$  states is reversed from  $^9\text{Be}$  to  $^{11}_{\Lambda\Lambda}\text{Be}$ . This is because the energy gain due to the addition of the  $\Lambda$ -particle(s) is larger in the compactly coupled state ( $5/2^-$ ) than in the loosely

coupled state ( $1/2_1^+$ ). Note that the same type of theoretical prediction was reported, in our early work [12] for  $^{13}\text{C}$  based on the  $\alpha\alpha\alpha\Lambda$  model, for the  $\Lambda$  particle addition to the compactly coupled state ( $3_1^-$ ) and to the loosely coupled state ( $0_2^+$ ) in  $^{12}\text{C}$ .

As seen in Fig. 3 the calculated value of  $B_{\Lambda\Lambda}(^{11}_{\Lambda\Lambda}\text{Be})$  is 18.23 MeV for the  $3/2^-$  ground state, while for the excited states the  $B_{\Lambda\Lambda}$  values are calculated to be less than 15.5 MeV. Therefore, the observed Hida event can be interpreted to be the ground state. When our calculated binding energy is compared with the experimental value of 20.83 MeV with a large uncertainty of  $\sigma=1.27$  MeV, we can say at least that our result does not contradict the data within  $2\sigma$ .

So far, the  $\Lambda\Lambda$  interaction strength has been often estimated rather intuitively by the quantity  $\Delta B_{\Lambda\Lambda}(\Lambda Z) \equiv B_{\Lambda\Lambda}(\Lambda Z) - 2B_{\Lambda}(\Lambda Z)$ . The observed value of  $\Delta B_{\Lambda\Lambda}(^6_{\Lambda\Lambda}\text{He})$  is 0.67 MeV, to which our  $\Lambda\Lambda$  interaction is adjusted. The calculated value for the ground state of  $^{10}_{\Lambda\Lambda}\text{Be}$  is 1.32 MeV. On the other hand, that for  $^{11}_{\Lambda\Lambda}\text{Be}$  is obtained as only 0.29 MeV. Here, the  $B_{\Lambda}(^1_0\text{Be})$  is given by a weighted sum of the values for ground  $1^-$  and excited  $2^-$  states of  $^1_0\text{Be}$  so that there is no contribution of the  $\Lambda N$  spin-spin interaction in the double- $\Lambda$  state. One should notice here the remarkable difference between  $\Delta B_{\Lambda\Lambda}$  values for  $^{10}_{\Lambda\Lambda}\text{Be}$  and  $^{11}_{\Lambda\Lambda}\text{Be}$ .

Furthermore, as discussed in the previous paper [6], values of  $\Delta B_{\Lambda\Lambda}$  include rearrangement effects in nuclear cores due to participation of  $\Lambda$  hyperons. Then, we showed that the  $V_{\Lambda\Lambda}^{\text{bond}}$  defined by Eq. (3) was useful for estimating the strength of the  $\Lambda\Lambda$  interaction

$$V_{\Lambda\Lambda}^{\text{bond}}(\Lambda Z) \equiv B_{\Lambda\Lambda}(\Lambda Z) - B_{\Lambda\Lambda}(\Lambda Z : V_{\Lambda\Lambda} = 0), \quad (3)$$

where  $B_{\Lambda\Lambda}(\Lambda Z : V_{\Lambda\Lambda} = 0)$  denotes the  $B_{\Lambda\Lambda}$  value calculated by putting  $V_{\Lambda\Lambda} = 0$ . Table II lists the calculated values of  $V_{\Lambda\Lambda}^{\text{bond}}$  for  $^6_{\Lambda\Lambda}\text{He}$ ,  $^{10}_{\Lambda\Lambda}\text{Be}$  and  $^{11}_{\Lambda\Lambda}\text{Be}$  which are similar to each other. Thus, the obtained  $\Lambda\Lambda$  bond energy in  $^{11}_{\Lambda\Lambda}\text{Be}$  turns out to be reasonable in spite of the small value of  $\Delta B_{\Lambda\Lambda}$ .

TABLE II:  $\Lambda\Lambda$  bond energy  $V_{\Lambda\Lambda}^{\text{bond}}(\Lambda Z)$  defined by Eq. (3).

	$^6_{\Lambda\Lambda}\text{He}$	$^{10}_{\Lambda\Lambda}\text{Be}$	$^{11}_{\Lambda\Lambda}\text{Be}$	
$V_{\Lambda\Lambda}^{\text{bond}}(\Lambda Z)$	0.54	0.53	0.56	(MeV)

In conclusion, motivated by the recent observation of the Hida event for a new double  $\Lambda$  hypernucleus, we have succeeded in performing a five-body calculation of  $^{11}_{\Lambda\Lambda}\text{Be}$  using an  $\alpha\alpha n\Lambda\Lambda$  cluster model. The calculated  $\Lambda\Lambda$  binding energy does not contradict the interpretation that the Hida event is an observation of the ground state of  $^{11}_{\Lambda\Lambda}\text{Be}$ . In this model, we described the core nucleus  $^9\text{Be}$  using the  $\alpha\alpha n$  three-cluster model and found a significant structure change (shrinkage) of  $^9\text{Be}$  due to the addition of the two  $\Lambda$  particles. For an alternative interpretation of the Hida event as the ground (or any excited) state of  $^{12}_{\Lambda\Lambda}\text{Be}$ , a corresponding six-body  $\alpha\alpha nn\Lambda\Lambda$  model calculation is necessary, but such an undertaking is beyond our present consideration. More precise data are needed in order to test our present result quantitatively. In the near future, many data for double  $\Lambda$  hypernuclei are expected to be found in the new emulsion experiment E07 at J-PARC. Then, our systematic predictions including the work of Ref. [6] will be clearly tested.

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  - [13] The  $B_{\Lambda\Lambda}$  values of NAGARA and Demachi-Yamanagi events are given as  $7.25 \pm 0.19$  MeV and  $12.33^{+0.35}_{-0.21}$  MeV in [1, 3] and [3], respectively. In Ref.[2], however, the E176/373 emulsion events have been reanalyzed by using the new  $\Xi^-$  mass [7]. Their newly-obtained values of  $B_{\Lambda\Lambda}$  for these events are shown in the text.